Paper Project

"On Krause's Multi-Agent Consensus Model with state-dependent connectivity"

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15. Juni 2012

Contents

Introduction

- Krause Model
- Building of groups (socalled clusters)
- Averaging the opinions with every timestep

Mathemathical Concepts

Krause Model Equation

$$x_i\left(t+1\right)=\frac{\sum_{j:\left|x_i(t)-x_j(t)\right|\prec 1} x_j\left(t\right)}{\sum_{j:\left|x_i(t)-x_j(t)\right|\prec 1} 1} \tag{1}$$

Assumption

1. *Motion of an agent depends on it's value and the value of the other agents it is connected with*
2. *Two agents keep their relativ position for all times*

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Characteristics

1. Two agents keep their relativ position for all times (Predifinition of order)
2. Granted convergence
3. Changing topology
4. Convergence to a stable point increases with the number of agents
5. Limited Convergence time

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Predefinition of order

Definitions

- Initialcondition $x(0) = [x_n(0) \cdots x_1(0)]$
- $\mathcal{N}_i \equiv$ set of agents connected to $x_i(0)$
- $\mathcal{N}_{ij} \equiv$ set of agents connected to $x_i(0)$ and $x_j(0)$
- $\mathcal{N}_j \equiv$ set of agents connected to $x_j(0)$
- $k_1 \in \mathcal{N}_i$
- $k_2 \in \mathcal{N}_{ij}$
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Predefinition of order

Assumptions

- $x_i(0) \prec x_j(0)$
- $x_{k_1}(t) \prec x_{k_2}(t) \prec x_{k_3}(t)$
- $\bar{x}_{N_i} \prec \bar{x}_{N_{ij}} \prec \bar{x}_{N_i}$

Conclusion

$$x_i(t+1) = \frac{|\mathcal{N}_{ij}|\bar{x}_{\mathcal{N}_{ij}} + |\mathcal{N}_i|\bar{x}_{\mathcal{N}_i}}{|\mathcal{N}_{ij}| + |\mathcal{N}_i|} \leq \bar{x}_{\mathcal{N}_{ij}}$$

$$x_j(t+1) = \frac{|\mathcal{N}_{ij}|\bar{x}_{\mathcal{N}_{ij}} + |\mathcal{N}_j|\bar{x}_{\mathcal{N}_j}}{|\mathcal{N}_{ij}| + |\mathcal{N}_j|} \geq \bar{x}_{\mathcal{N}_{ij}}$$

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Granted Convergence

Granted convergence

Convergence according to the Bolzano-Weierstrass theorem

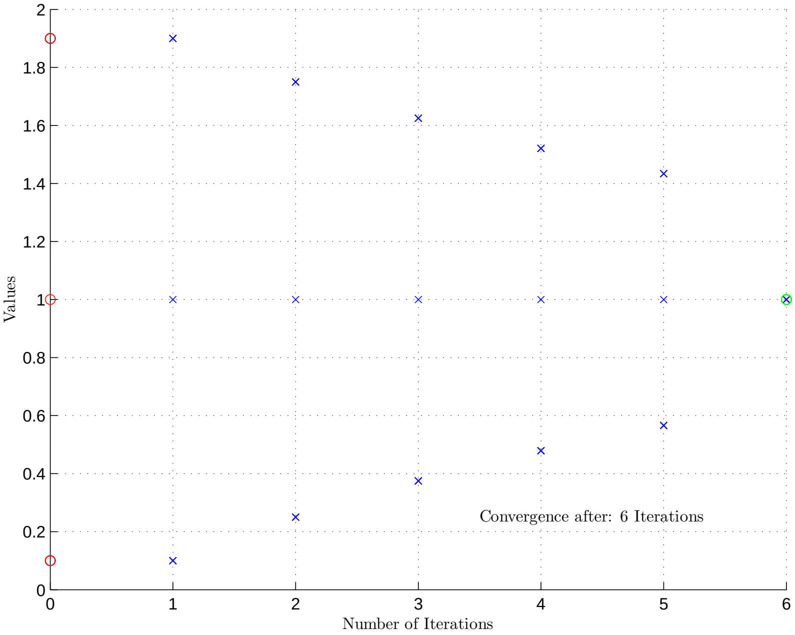
- Every element of $x(t)$ is bounded by $x_n(0)$ and $x_1(0)$
- $x_n(t)$ can only decrease and $x_1(t)$ can only increase (see previous slide)

Convergence to a stable point increases with the number of agents

Example

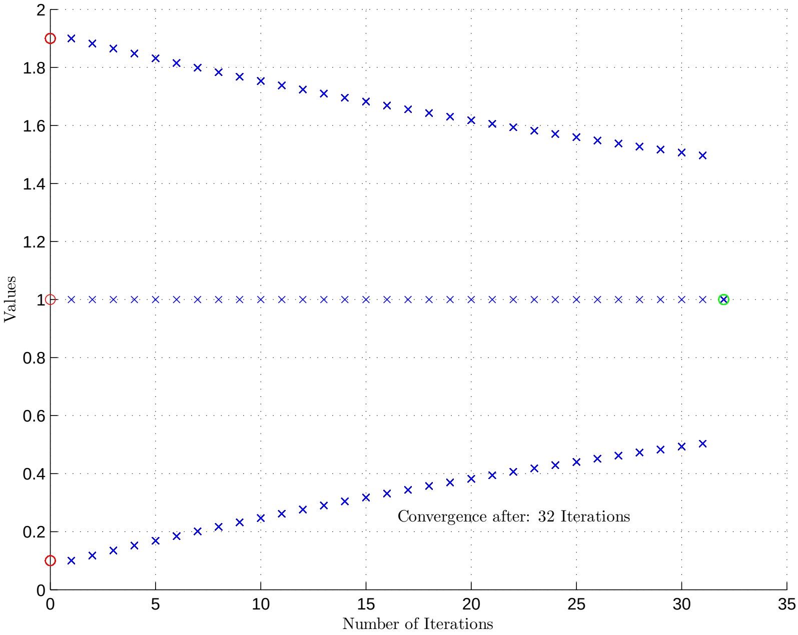
- Odd number of agents
- Distribution of
  - $\frac{n-1}{2}$ agents at 0.1
  - $\frac{n-1}{2}$ agents at 1.9
  - 1 agent at 1.0

11 agents



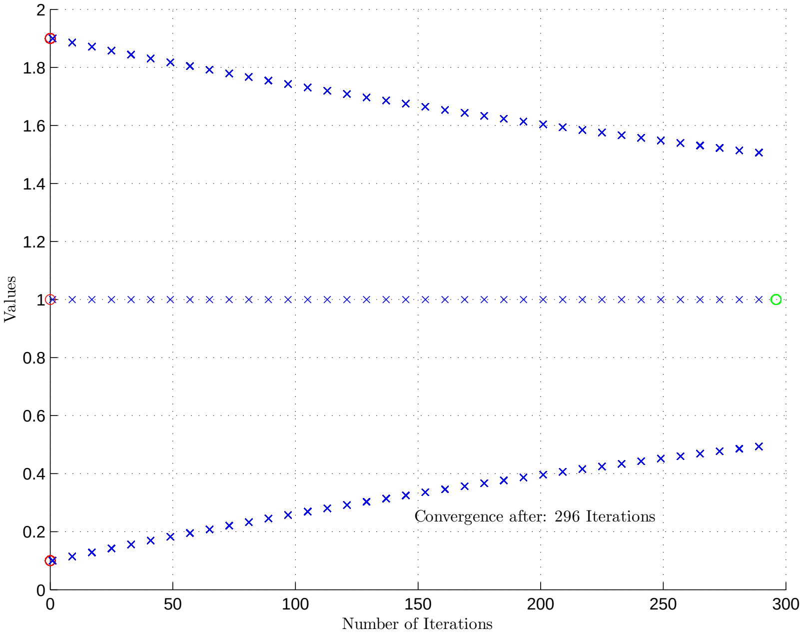
Values
Number of Iterations
Convergence after: 6 Iterations
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
1
2
3
4
5
6

101 agents



Values
Number of Iterations
Convergence after: 32 Iterations
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
5
10
15
20
25
30
35

1001 agents



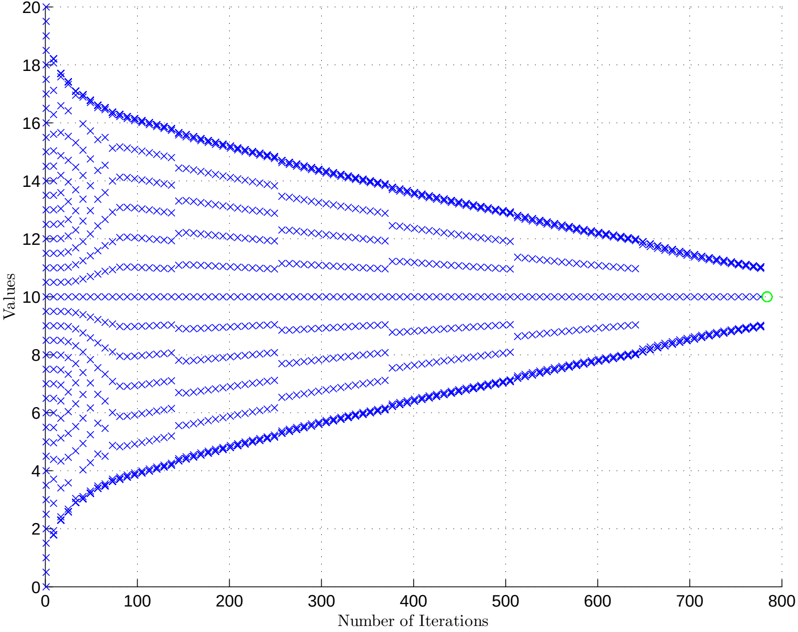
Values
Number of Iterations
Convergence after: 296 Iterations
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
50
100
150
200
250
300

Meta-stabile clusters

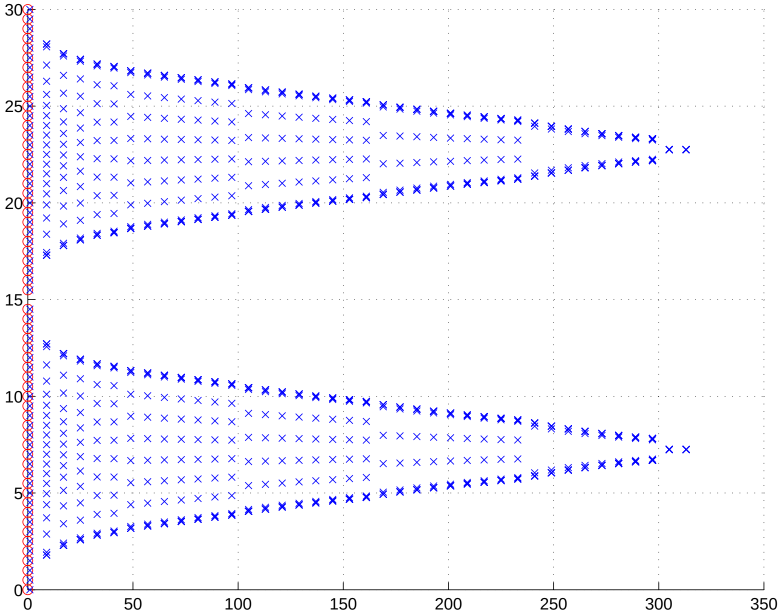
Conditions

- uniformly spaced agents
- density of 2 agents per length unit
- $\Rightarrow$ Initial distance of 0.5

40 agents + 1 agent in the middle



60 agents and no agent in the middle



Limited convergence time

If $x(t)$ developes according to (1) there is a time t' at which all agents get sufficiently close together so that all of them are connected together and reach the same average. Because of this they converge in finite time. Furthermore the system converges in finite time because the number of agents is finite, too.

Stability with respect to a pertubing agent

Assumptions

- Weights w_i
- (1) changes to

$$x_i\,(t+1) = \frac{\sum_{j:\left|x_i(t)-x_j(t)\right|\prec 1} w_j x_j\,(t)}{\sum_{j:\left|x_i(t)-x_j(t)\right|\prec 1} w_j} \qquad (2)$$

- Clusters A and B are weighted by W_A and W_B
- Added agent will be removed *after new equilibrium is reached*

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Stability with respect to a pertubing agent

Theorem

An equilibrium is stable if either $W_A = W_B$ and the distance between them is greater than or equal to 2 or if $W_A \neq W_B$ and the distance between them is *strictly* greater than $1 + \frac{\min(W_A, W_B)}{\max(W_A, W_B)}$